



**Oliy ta'lim muassasa talabalari uchun  
masofaviy olimpiada masalalari  
3-topshiriq (21.04.2017 – 10.05.2017)**

1. Agar  $n \in \mathbb{N}$  bo'lsa, u holda quyidagi tengliklarni isbotlang:

$$\int_0^{\frac{\pi}{2}} \frac{\sin(2n-1)x}{\sin x} dx = \frac{\pi}{2},$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin 2nx}{\sin x} dx = 2 \sum_{k=1}^n \frac{(-1)^{k-1}}{2k-1}.$$

2. Sirkul va chizg'ich yordamida teng tomonli uchburchakni shunday bo'laklarga bo'lingki, ulardan kvadrat hosil qilib bo'lsin.

3. Ixtiyoriy  $(a, b) \subset \mathbb{R}$  da kontinium uzulishga hamda kontinium uzluksiz nuqtaga ega bo'lgan  $f : \mathbb{R} \rightarrow \mathbb{R}$  funksiya quring.

4. Agar  $p > 3$  tub son hamda

$$\frac{n}{m} = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{p}$$

bo'lsa, u holda  $n - m$  qiymat  $p^3$  ga bo'linishini isbotlang.

5. Agar  $\{x_n\}$  ketma-ketlik uchun  $x_1 = 1$  va  $x_{n+1} = x_n + \frac{1}{x_n}$  shartlar bajarilsa,  $\lim_{n \rightarrow \infty} \frac{x_n}{\sqrt{n}}$  ni toping.

6. Agar  $f \in C^1[a, b]$  va

$$\int_a^b f(x) dx = \int_a^b f'(x) dx = 0$$

bo'lsa, u holda

$$|f(x)| \leq \frac{1}{2} \int_a^b |f'(x)| dx, \quad \forall x \in [a, b]$$

tengsizlikni isbotlang.